Effectiveness-*NTU* Relationships for Tubular Exchangers

E. M. Smith†

The use of expressions for effectiveness (ε) and number of transfer units (Ntu) minimizes design effort required in the sizing of heat exchangers. Kays and London (1) present results for a number of cases of practical importance. Further examples of ε-Ntu expressions for contra-parallel flow and for bayonet tube exchangers which do not appear to have been published, and which may be added to the collection, are given below.

NOMENCLATURE

c_p, C_p	specific heat
h, H	overall heat transfer coefficient
$\dot{m}$, $\dot{M}$	mass flow rate
ntu, NTU	number of transfer units
s, S	total surface area
t, T	temperature
W	thermal ratio (defined in Figs. 1 and 2)
x	length
α, β, Γ, Δ	expressions defined in Figs. 1 and 2
ε	effectiveness
$\Delta\theta_m$	log mean temperature difference (LMTD)
θ	temperature

Subscripts

o, i	outlet, inlet
1, 2	first, second
c, pc	contraflow, parallel-contraflow
m	middle

INTRODUCTION

For shell fluid mixed, the parallel-contraflow LMTD expression in terms of terminal temperatures was first presented by Underwood (2) and obtained in ε-Ntu form by Wright (3). Assumptions made in the Underwood analysis are listed by Jakob (4) as follows:

(1) Steady-state conditions prevail for flow of fluids and temperature
(2) Changes of state (vaporization, condensation, etc.) do not occur
(3) Each fluid is so well mixed that its temperature is constant over the cross-section of flow
(4) The specific heats of the two fluids are constant
(5) The overall coefficient H is independent of place and temperature
(6) The heating surface is the same in each pass
(7) Heat losses are negligible

The above assumptions apply also to new results (Fig. 1) obtained for contra-parallel flow and for the four cases of bayonet tube exchangers (Fig. 2). Results for parallel-contra flow derived earlier by Wright are included in Fig. 1 for completeness.

† Department of Mechanical Engineering, University of Newcastle upon Tyne, NE1 7RU.

Received 23 August 1978 and accepted for publication on 23 January 1979.

ILLUSTRATIVE EXAMPLE (PARALLEL-CONTRAFLOW)

Figure 3 depicts a proposed organic fluid heater directly fired by natural gas. The problem is to prevent overheating and decomposition of the organic fluid, while recovering the maximum amount of energy from combustion products.

Figure 4 illustrates the design concept. To limit outlet temperature of the organic fluid, a parallel contraflow pass is used to obtain the lowest exhaust gas temperature.

It is essential to check the temperature of the organic fluid at the tube wall to assess the degree of decomposition likely to occur. Before this can be done, bulk fluid temperatures have to be determined.

Relationships governing heat transfer are as follows

Contraflow pass	*Parallel-contraflow pass*
$\varepsilon_c = \dfrac{t_2 - t_3}{t_2 - T_1}$	$\varepsilon_{pc} = \dfrac{t_1 - t_2}{t_1 - T_2}$
$W_c(t_2 - t_3) = T_2 - T_1$	$W_{pc}(t_1 - t_2) = T_3 - T_2$

The solution of these four equations is presented in Table 1.

For the configuration shown in Fig. 3 values of NTU_c, W_c and NTU_{pc}, W_{pc} are known for any particular heater design. Numerical values for ε_c and ε_{pc} may then be calculated from

$$\varepsilon_c = \frac{1 - \exp\left[-NTU_c(1 - W_c)\right]}{1 - W_c \exp\left[-NTU_c(1 - W_c)\right]}$$

(standard result)

$$\varepsilon_{pc} = \frac{2}{1 + W_{pc} + \sqrt{(1 + W_{pc}^2)}\,[(1 + e^{-\Gamma})/(1 - e^{-\Gamma})]}$$

where $\Gamma = NTU_{pc}\sqrt{(1 + W_{pc}^2)}$.

Given combustion gas and organic fluid bulk entry temperatures t_1 and T_1, bulk exit temperatures t_3, T_3 and bulk intermediate temperatures t_2, T_2 may be determined using equations from Table 1.

(7) gives $t_2 = \dfrac{t_1[1 - (1/\varepsilon_{pc})] - T_1 + W_c\varepsilon_c T_1}{W_c\varepsilon_c - (1/\varepsilon_{pc})}$

(1) gives $t_3 = \varepsilon_c T_1 + (1 - \varepsilon_c)t_2$

(6) gives $T_3 = (W_c - W_{pc})t_2 - W_c t_3 + W_{pc} t_1 + T_1$

(2) gives $T_2 = \dfrac{1}{\varepsilon_{pc}} t_2 + \left(1 - \dfrac{1}{\varepsilon_{pc}}\right) t_1$

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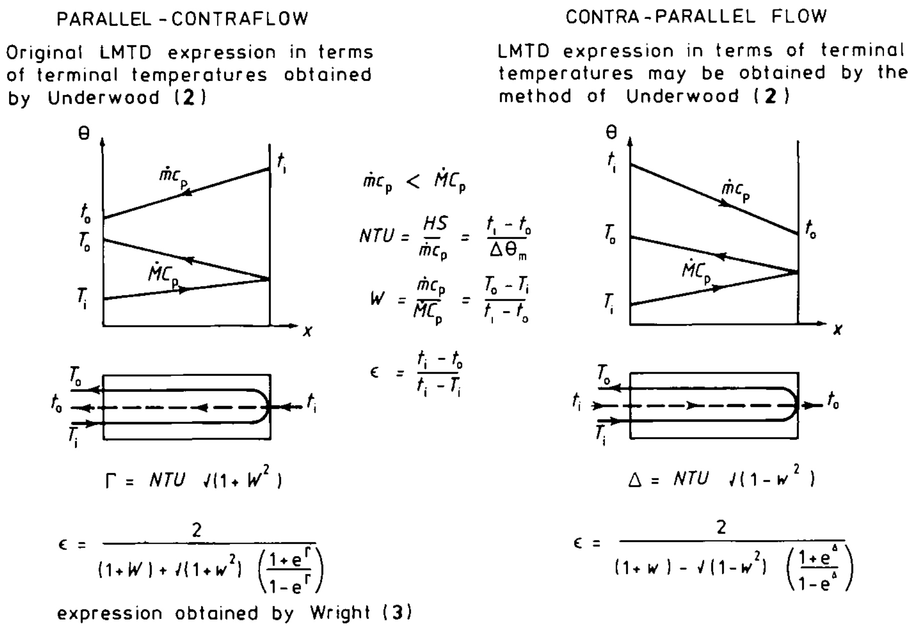

Fig. 1. Crossflow, shell-side fluid mixed

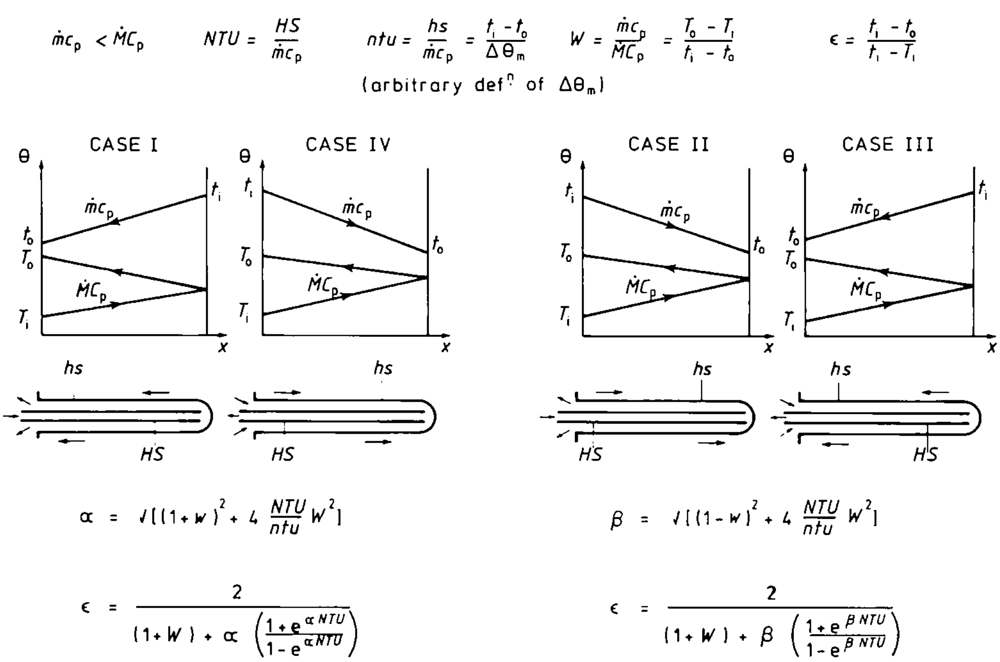

Fig. 2. Bayonet tube exchangers. Original LMTD expressions in terms of terminal temperatures obtained by Hurd (**5**)

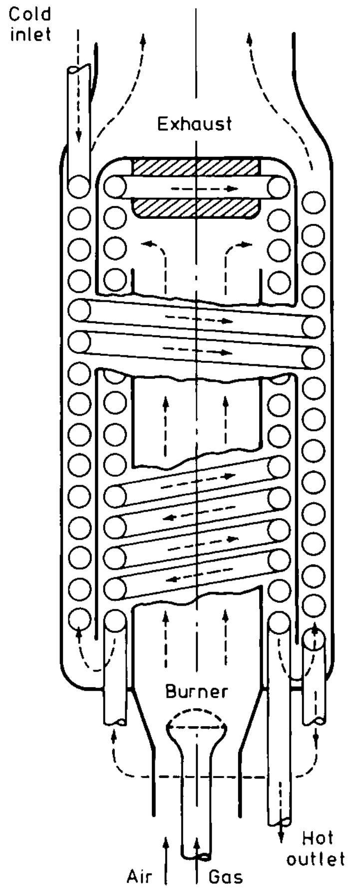

Fig. 3. Direct fired organic fluid heater

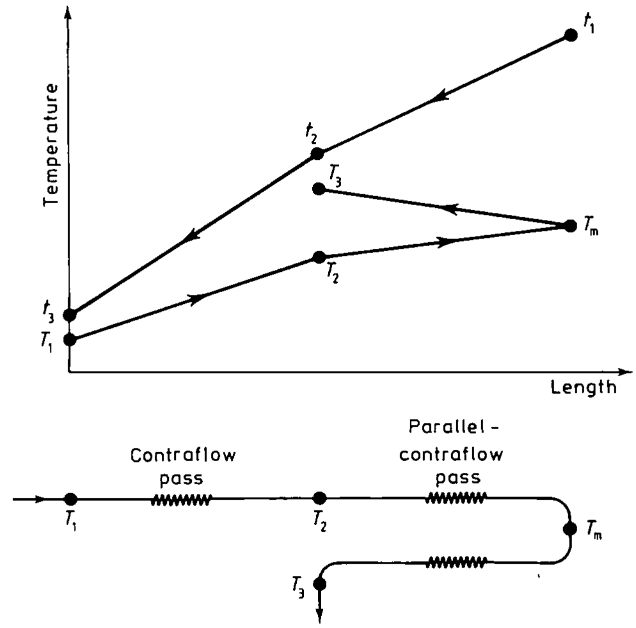

Fig. 4. Temperature profiles (schematic) for direct fired organic fluid heater

Middle temperature T_m for the parallel-contraflow pass (Fig. 4) is

$$T_m = t_i - \frac{\sqrt{(1 + W_{pc}^2)}(t_1 - t_2) \exp (NTU_{pc}/2)}{2 \sinh (\Gamma/2)}$$

Temperature drops across boundary layers may now be evaluated and wall temperatures determined at specified points.

Note that the temperature profiles depicted in Figs. 1, 2, and 4 will not necessarily be straight even when physical properties of both fluids do not vary with temperature. If more precise knowledge of conditions along each pass is required, then it would be necessary to return to the solution of the differential equations quoted in the original papers by Underwood (**2**) and Hurd (**5**) which provide the desired relationships.

Usually it is possible to make reasonable engineering decisions without resort to such detailed analysis, but special care must be exercised if physical properties of either single-phase fluid vary significantly with temperature. In particular, results in this paper should not be used when phase changes are encountered.

BAYONET HEAT EXCHANGERS

These are sometimes of interest in the chemical industry when corrosion-resistant metals may only be available in straight tubular form. Figure 5 illustrates a design to

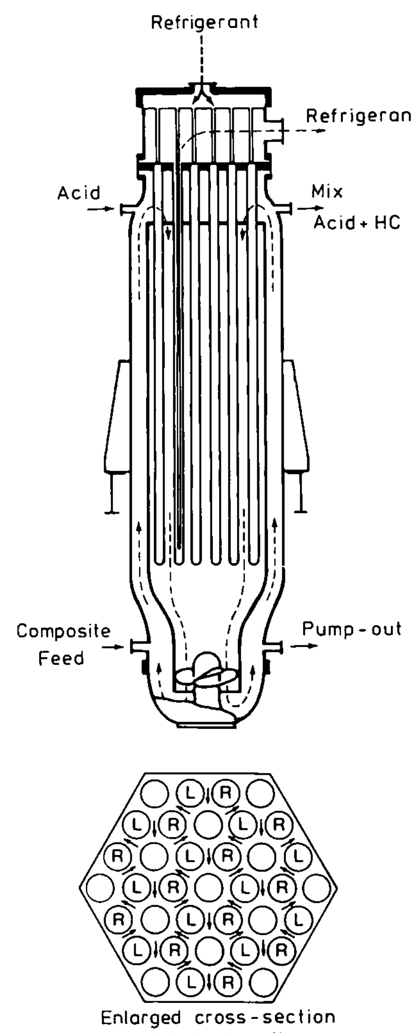

Fig. 5. Bayonet tube exchanger after Hurd (**5**)

Table 1

Solution of Terminal Temperatures

Equation	Unknown temperatures t_2	t_3	T_2	T_3	RHS
(1)	$(1-\varepsilon_c)$	-1			$-\varepsilon_c T_1$
(2)	-1		ε_{pc}		$t_1(\varepsilon_{pc}-1)$
(3)	W_c	$-W_c$	-1		$-T_1$
(4)	$-W_{pc}$		1	-1	$-W_{pc}t_1$
(1)	$(1-\varepsilon_c)$	-1			$-\varepsilon_c T_1$
$(2) \div \varepsilon_{pc} = (2)$	$-(1/\varepsilon_{pc})$		1		$t_1(1-1/\varepsilon_{pc})$
(3)	W_c	$-W_c$	-1		$-T_1$
(4)	$-W_{pc}$		1	-1	$-W_{pc}t_1$
(1)	$(1-\varepsilon_c)$	-1			$-\varepsilon_c T_1$
$(2)+(3)=(5)$	(W_c-1/ε_{pc})	$-W_c$			$t_1(1-1/\varepsilon_{pc})-T_1$
$(3)+(4)=(6)$	(W_c-W_{pc})	$-W_c$		-1	$-W_{pc}t_1-T_1$
(1)	$(1-\varepsilon_c)$	-1			$-\varepsilon_c T_1$
$(5) \div W_c = (5)$	$(1-1/W_c\varepsilon_{pc})$	-1			$[t_1(1-1/\varepsilon_{pc})-T_1]/W_c$
(6)	(W_c-W_{pc})	$-W_c$		-1	$-W_{pc}t_1-T_1$
$(5)-(1)=(7)$	$(1-1/W_c\varepsilon_{pc})-(1-\varepsilon_c)$				$[t_1(1-1/\varepsilon_{pc})-T_1]/W_c+\varepsilon_c T_1$

which ε-*Ntu* relationships presented earlier may be applied.

Heat transfer on the shell-side of such exchangers may be enhanced if some of the tubes are provided with right-hand and left-hand slow spiral wire wraps, and some are left plain.

Cautionary Note

With the bayonet and hairpin tube configurations depicted, temperature differences at each end of the exchanger should be checked to ensure that temperature cross-overs do not occur. For the design shown in Figs. 2 and 3, since t_2 and T_3 are expressible in terms of t_1 and T_1, the constraint $t_2 - T_3 > 0$ can and should be tested, since solution of eqs. (1)–(4) does not ensure this condition.

ACKNOWLEDGEMENT

The illustrative example is included with kind permission of the management of the Glynwed CRU organization, for whom the work was done.

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